Ordinary Differential Equation

Definition
Order of Diff. Eq.
Degree of Diff Eq.
} Already done in class.

Example. Find order and degree for the equations:

1) $\underset{\text{I}}{\frac{d^2y}{dx^2}} + 2\underset{\text{II}}{\left(\frac{dy}{dx}\right)^3} + \underset{\text{III}}{2y} = 0$

order → is highest order derivative

here, highest order derivative is 2

∴ order is 2

degree → is degree of highest order derivative

here, I term is highest order derivative and its degree is 1

∴ degree is 1.

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Remark: To find order and degree of D.E. (differential eqn), all the derivatives should be free from any fraction or radical form.

for example.

(a) $$\frac{d^2y}{dx^2} = 1 + \sqrt{\frac{dy}{dx}}$$

$$\frac{d^2y}{dx^2} = 1 + \left(\frac{dy}{dx}\right)^{1/2}$$

here, $\frac{dy}{dx}$ → is a derivative term and its power is in radical form

(radical means : root of something)

So we need to remove this power $1/2$

$$\Rightarrow \frac{d^2y}{dx^2} - 1 = \left(\frac{dy}{dx}\right)^{1/2}$$

Now square both side

$$\left(\frac{d^2y}{dx^2} - 1\right)^2 = \frac{dy}{dx}$$

(3) $(a-b)^2 = a^2 + b^2 - 2ab$

$$\left(\frac{d^2y}{dx^2}\right)^2 + 1 - 2\frac{d^2y}{dx^2} = \frac{dy}{dx}$$

Now, there is no power in root form.

So here, order is 2 (highest order derivative)

and degree is 2

(3) $$y = \underset{\text{I}}{\frac{dy}{dx}} + \underset{\text{III}}{\frac{c}{dy/dx}}$$

here, $\frac{dy}{dx}$ is in fraction $\frac{c}{dy/dx}$ (in III term). So we need to modify to remove this fraction

$$\Rightarrow y = \frac{dy}{dx} + \frac{c}{dy/dx}$$

Take L.C.M.

$$y = \frac{(dy/dx)^2 + c}{dy/dx}$$

$$y \cdot \frac{dy}{dx} = \left(\frac{dy}{dx}\right)^2 + c$$

order = 1 (highest order deri).

degree = 2

(4)

(4) $\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \left(c\,\frac{d^2y}{dx^2}\right)^{1/3}$

or $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{1/2} = \left(c\,\frac{d^2y}{dx^2}\right)^{1/3}$

here, first we need to remove the power $1/2$ and $1/3$

$\Rightarrow$ first to remove the power $1/2$.

$\therefore$ square on both side

$$\left[1+\left(\frac{dy}{dx}\right)^2\right] = \left(c\,\frac{d^2y}{dx^2}\right)^{2/3}$$

Now, take the cube on both side

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \left(c\,\frac{d^2y}{dx^2}\right)^2$$

$$(a+b)^3 = a^3+b^3+3a^2b+3ab^2$$

$$1+\left(\frac{dy}{dx}\right)^6+3\left(\frac{dy}{dx}\right)^2+3\left(\frac{dy}{dx}\right)^4 = c^2\left(\frac{d^2y}{dx^2}\right)^2$$

(*)

$\Rightarrow$ Order = 2 (* term)

degree = 2.

(Do it yourself)

Que: Evaluate the order and degree.

(1) $\left(\frac{dy}{dx}\right)^2 + \frac{1}{dy/dx} = 2$

(2) $\frac{d^4y}{dx^4} = \left[c + \left(\frac{dy}{dx}\right)^2\right]^{3/2}$

(3) $\frac{d^3y}{dx^3} + \left(\frac{d^2y}{dx^2}\right)^3 + \frac{dy}{dx} + 4y = \sin x$

(4) $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$

(5) $\left(\frac{dy}{dx}\right)^4 + 3y\left(\frac{d^2y}{dx^2}\right) = 0$

Linear And Non Linear Equation

Linear: A D.E. is said to be linear if

(i) dependant variable (means y) and its derivative (means form of $\frac{dy}{dx}$) have only first degree

(ii) and y and any form of $\frac{dy}{dx}$ should Not be multiplied together.

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Meaning of these two points :

(i) dependent variable (means y) and its derivative (any form of dy/dx) should be occur only in first degree.

→ there should be No $y^2, y^3 \ldots$ (any power of y) in given eqn

y must have only degree 1

for eg. $\frac{d^2y}{dx^2} + y^2 = 0$

y^2 ✗ (Power is 2 of y)

⇒ Not Linear.

→ Same for any form of $\frac{dy}{dx}$.

All form of $\frac{dy}{dx}$ must have degree 1.

for eg. $\frac{dy}{dx}$ ✓, $\frac{d^2y}{dx^2}$ ✓, $\left(\frac{dy}{dx}\right)^2$ ✗, $\frac{d^3y}{dx^3}$ ✓,

$\left(\frac{d^2y}{dx^2}\right)^2$ ✗

✓ → for having degree 1

✗ → degree is Not 1

(7)

(ii) Point says that y and $\frac{dy}{dx}$ should not be multiplied together.

for eg. (a) $y \cdot \frac{dy}{dx} + \sin x = 0$

y and $\frac{dy}{dx}$ are in multiply

$\Rightarrow$ Not Linear.

(b) $y \cdot \frac{d^2y}{dx^2} + \sin x = 0$

y and $\frac{d^2y}{dx^2}$ are in Multiply

$\Rightarrow$ Not Linear.

NOTE: For any Equation to be Linear both the points Must satisfy.

$\rightarrow$ If one point satisfy and other is Not then given equation will Not be LINEAR

Non Linear: Any D.E which is Not Linear then it is Non-Linear. (i.e which Not satisfy these two points)

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eg: (1) $\frac{dy}{dx} = \sin x$

here, degree of $\frac{dy}{dx}$ is 1

$\Rightarrow$ it is Linear. (Both point satisfy)

(2) $\frac{dy}{dx} + \frac{2}{x} y = x^3$

here, $\frac{dy}{dx}$ has degree 1 and

Power of y is also 1

$\Rightarrow$ it is linear.

(3) $y \cdot \frac{dy}{dx} = \left(\frac{dy}{dx}\right)^2 + c$

here $y \cdot \frac{dy}{dx}$, $\rightarrow$ are in Multiply

Also degree of $\frac{dy}{dx}$ is 2

Both the condition Not satisfied

$\Rightarrow$ Non - Linear.

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(4) $\underset{\text{I}}{\frac{d^2y}{dx^2}} + \underset{\text{II}}{2\left(\frac{dy}{dx}\right)^3} + \underset{\text{III}}{2y} = 0$

Order for any $\frac{dy}{dx}$ can be 1, 2, 3, ...

BUT degree must be 1.

I term $\frac{d^2y}{dx^2}$ → degree is 1.

II term $2\left(\frac{dy}{dx}\right)^3$ → degree is 3 (Not 1)

⇒ Ist condition is Not satisfied

⇒ Non - Linear.

Que: (Do it yourself)
Check the given equations are Linear or Not Linear on Page (5)